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A FLOW OF A HEAT-CONDUCTING GAS ANALOGOUS TO A CENTRED RIEMANN WAVE†

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A flow of a heat-conducting non-viscous gas, analogous to a centred Riemann wave and transmitting an intense compression of one-dimensional layers of gas is described using a special infinite converging series, taking a number of physical effects into account.

To describe certain plane-symmetrical gas flows, a centred Riemann wave is used – the solution of the system of equations of gas dynamics possessing a specific characteristic: $\mathbf{U} = \mathbf{U}(x_1/t)$, where $\mathbf{U}$ is the vector of the required functions. Many interesting problems in gas dynamics can be solved using the centred Riemann wave [1]. In the case of cylindrically and spherically symmetrical flows the self-similar solutions possess a similar feature: $\mathbf{U} = \mathbf{U}(r/t)$. Using these solutions one can describe, in particular, the unbounded and bounded contraction of a gas to the axis or the centre of symmetry [3, 4]. To describe a flow possessing the properties of a centred Riemann wave in the neighbourhood of a point with a non-zero value of the coordinate r (for example, $r = 1$) or in the multidimensional case, special converging infinite series are employed (for a detailed bibliography see [5]). The following problems, for example, have been solved using such series: the instantaneous stopping of a piston [6], the escape of a gas into a vacuum [7–9], and the shock-free intense compression of a gas [5, 10]. In problems of the intense compression of a gas, in order to provide a more adequate description of the flows that occur it is necessary to take into account the equilibrium radiation and the Compton mechanism of the scattering of photons [11, 12].

1. CONSTRUCTION OF THE SERIES

We will consider an ideal gas, taking into account the equilibrium radiation, i.e. we will take the following relations as the equations of state [12]

$$p = R\rho T + \sigma T^4/3, \quad e = c_v T + \sigma T^4/\rho; \quad R, \sigma, c_v = \text{const} > 0 \tag{1.1}$$

Here p is the pressure, ρ is the density, T is the temperature, e is the internal energy and σ is the Stefan–Boltzmann constant.

The thermal conductivity $\varkappa$, consistent with the Compton mechanism of photon scattering, is taken as [12]

$$\varkappa = \frac{2}{\gamma - 1}\sigma c_* \alpha \frac{T^3}{\rho}; \quad \gamma - 1 = \frac{R}{c_v} > 0 \tag{1.2}$$

where c_* is the velocity of light and α is a positive constant, which depends on the choice of the system of units.

To describe the flows of such a gas we can take ρ and T to be independent thermodynamic variables. Hence, we can use [13] the complete system of Navier–Stokes equations, in which the coefficients of dynamic and volume viscosity are assumed to be equal to zero. In the case of one-dimensional flows ($\nu = 0, 1, 2$ corresponds to plane, cylindrical and spherical symmetry) and introducing $\vartheta = \ln \rho$ as the required function instead of the density, this system has the form

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$$\vartheta_t + u\vartheta_r + u_r + \nu r^{-1}u = 0$$
$$u_t + uu_r + \gamma^{-1}T\vartheta_r + a(\vartheta,T)T_r = 0 \tag{1.3}$$
$$T_t + uT_r + b(\vartheta,T)(u_r + \nu r^{-1}u) = \varkappa_* c(\vartheta,T)(T_{rr} + \nu r^{-1}T_r - \vartheta_r T_r + 3T_r^2 / T)$$
$$a(\vartheta,T) = \gamma^{-1}(1 + \varkappa_* k_1 e^{-\vartheta}T^3), \qquad b(\vartheta,T) = (\gamma-1)T\frac{1+\varkappa_* k_1 e^{-\vartheta}T^3}{1+\varkappa_* k_2 e^{-\vartheta}T^3}$$
$$c(\vartheta,T) = \frac{e^{-2\vartheta}T^3}{1+\varkappa_* k_2 e^{-\vartheta}T^3}$$

In the system considered, without loss of generality, we can take the value $t = 0$ as the time origin. Also, using the positive constants L, ρ_* and T_* we can introduce, in a standard way, dimensionless variables, where the velocity of sound in a non-heat-conducting gas $u_* = \sqrt{R\gamma T_*}$ is taken as the velocity scale. Then

$$\varkappa_* = 2\sigma c_*\alpha\frac{T_*^3}{R\rho_*^2 u_* L} > 0, \qquad k_1 = \frac{2\rho_* u_* L}{3c_*\alpha} > 0, \qquad k_2 = 3(\gamma-1)k_1 > 0$$

To describe the features that arise in the flow of gas at the instant of intense compression, as in the case of a non-heat-conducting gas [5], we changed the roles of the variables ϑ and r. The variable ϑ (together with t) is assumed to be the independent variable, while r becomes the required function of t and ϑ. Hence, we make the following replacement of variables

$$t = t', \qquad r = r(t', \vartheta)$$

with the Jacobian transformation $J = -r_\vartheta$, if the value of $r_{t'}$ is finite.

The Jacobian J vanishes if, in the physical space (the space of the independent variables t and r), the derivative ϑ_r takes an infinite value. In this case, in the space of the independent variables t', ϑ the derivative of r_ϑ is equal to zero and there are no singularities. This property is the main reason for using this replacement: the required solution in the space of the independent variables t', ϑ at the point ($t' = 0$, $\vartheta = 0$) will have no singularities, and in the physical space at the corresponding point the gradient will have an infinite value. The unknown function $r(t', \vartheta)$, which is determined only when constructing the required solution, occurs in the proposed replacement, and therefore nothing is so far known about the quantity $r_\vartheta(t', \vartheta)$. After the corresponding problem is solved, and, consequently, the function $r(t', \vartheta)$ has been obtained, it turns out that $r_\vartheta(t', \vartheta)|_{t'=0} = 0$ and, for a specified value of ϑ_0, a $t_0 > 0$ will exist such that when $|\vartheta| < \vartheta_0$, $|t| < t_0$, $t \neq 0$ the inequality $r_\vartheta(t', \vartheta) \neq 0$ will hold.

When the above replacement is made, system (1.3) take the following form (we will henceforth omit the prime on t)

$$r(u - r_t) + ru_\vartheta + \nu ur_\vartheta = 0$$
$$r_\vartheta u_t + (u - r_t)u_\vartheta + \gamma^{-1}T + a(\vartheta,T)T_\vartheta = 0 \tag{1.4}$$
$$r_\vartheta^2[r_\vartheta T_t + (u - r_t)T_\vartheta + b(\vartheta,T)(u_\vartheta + \nu r^{-1}ur_\vartheta)] =$$
$$= \varkappa_* c(\vartheta,T)(r_\vartheta T_{\vartheta\vartheta} - r_{\vartheta\vartheta}T_\vartheta + \nu r^{-1}r_\vartheta^2 T_\vartheta - r_\vartheta T_\vartheta + 3r_\vartheta T_\vartheta^2 / T)$$

In this case, in order to expand the singularity, related to the presence of r_ϑ in the denominators in certain fractions, when obtaining system (1.4) the first equation is multiplied by rr_ϑ, the second by r_ϑ^2 and the third by r_ϑ^3.

The solution of system (1.4) is constructed in the form of a series

$$\mathbf{U}(t,\vartheta) = \sum_{k=0}^{\infty}\mathbf{U}_k(\vartheta)\frac{t^k}{k!}, \qquad \mathbf{U}_k(\vartheta) = \left.\frac{\partial^k \mathbf{U}(t,\vartheta)}{\partial t^k}\right|_{t=0} \tag{1.5}$$

where $\mathbf{U} = \{r, u, T\}$ is the vector of the required functions.

In order for the required flow at the point ($t = 0, r = 1$) to possess a singularity, similar to the singularity on the centred Reimann wave, the graph of the function $\vartheta = \vartheta(t, r)|_{t=\text{const}}$ when $t \to -0$ should become a vertical straight line [5]

$$r|_{t=0} = 1 \tag{1.6}$$

The solution of system (1.4) in the form of series (1.5) will for the present be constructed using one condition – condition (1.6).

If we put $t = 0$ in system (1.4) and the take equality (1.6) into account, the third equation becomes an identity, while the first two become the following relations

$$u_0 - r_1 + u_0' = 0, \quad (u_0 - r_1)u_0' + \gamma^{-1}T_0 + a(\vartheta, T_0)T_0' = 0 \tag{1.7}$$

If the third equation of system (1.4) is differentiated with respect to t, and if we put $t = 0$ and take condition (1.6) into account, we obtain the equation

$$r_1'T_0'' - r_1''T_0' - r_1'T_0' + 3r_1'T_0'^2/T = 0 \tag{1.8}$$

As a result, for the three required quantities r_1, u_0 and T_0 we obtain system (1.7), (1.8) from the three differential equation, two of which are non-linear. It is a quite difficult problem to find the general solution of this system, although some progress in this direction is possible: by dividing Eq. (1.8) by $r_1'T_0'$

$$(\ln T_0')' - (\ln r_1')' - 1 + 3(\ln T_0)' = 0$$

we can reduce the order of this equation

$$T_0^3 T_0' = C_1 e^{\vartheta} r_1', \qquad C_1 = \text{const}$$

Further, we will take as the function $T_0(\vartheta)$ the constant

$$T_0(\vartheta) = T_{00} = \text{const} > 0 \tag{1.9}$$

which converts Eq. (1.8) into an identity. The use of the particular solution (1.9) results from the following. It is precisely this property of the temperature – the absence of a jump in the function T at the instant $t = 0$ – that is observed in numerical calculations of the corresponding compression wave in a heat-conduction gas in the plane-symmetrical case [12].

In the case of the particular solution (1.9), the two remaining equations – Eqs (1.7) – have the following general solution

$$u_0(\vartheta) = \pm\sqrt{\gamma^{-1}T_{00}}\,\vartheta + u_{00}, \quad r_1(\vartheta) = \pm\sqrt{\gamma^{-1}T_{00}}\,\vartheta + u_{00} \pm \sqrt{\gamma^{-1}T_{00}} \tag{1.10}$$

In relations (1.9) and (1.10), as a result of integration, two arbitrary constants T_{00} and u_{00} appeared, while the function $r_1(\vartheta)$ was defined uniquely.

In order to obtain the coefficients r_{k+1}, u_k, and T_k when $k \geqslant 1$, it is necessary to differentiate the first two equations of system (1.4) with respect to t, k times and the third equation of (1.4) $k + 1$ times, and then put $t = 0$. As a result we obtain three equations

$$u_k - r_{k+1} + u_k' = F_k$$

$$kr_1'u_k + (u_0 - r_1)u_k' + u_0'(u_k - r_{k+1}) + \gamma^{-1}T_k + a(\vartheta, T_{00})T_k' = G_k$$

$$\varkappa_* T_{00}(k+1)r_1'(T_k'' - T_k') = H_k$$

where

$$F_k = -\sum_{l=1}^{k} C_k^l [r_l(u_{k-l} - r_{k-l+1} + u_{k-l}') + \nu r_l' u_{k-l}]$$

while the functions G_k and H_k (not given here because of their complexity) also depend on r_{l+1}, u_l, T_l($0 \leqslant l \leqslant k - 1$) and their derivatives.

From the third (differential) equation we first determine T_k

$$T_k = T_{k0} + T_{k1}e^{\vartheta} - a_k \int H_k d\vartheta + a_k e^{\vartheta} \int H_k e^{-\vartheta} d\vartheta \tag{1.11}$$

$$a_k = \pm \frac{\sqrt{\gamma}}{(k+1)\varkappa_* \sqrt{T_{00}^3}}$$

Then, from the second equation (also differential), first eliminating r_{k+1} from it using the first equation, we obtain u_k

$$u_k = u_{k0}e^{k\vartheta/2} + \frac{e^{k\vartheta/2}}{2} \int \left[\mp \sqrt{\frac{\gamma}{T_{00}}} G_k - F_k \right] e^{-k\vartheta/2} d\vartheta \tag{1.12}$$

Finally, from the first equation (as from the algebraic equation) we determine r_{k+1}

$$r_{k+1} = u_k' + u_k - F_k \tag{1.13}$$

Here, two arbitrary constants T_{k0} and T_{k1} occur in the expression for T_k; in u_k there is one constant u_{k0}, which occurs as a result of integrating the differential equations. The coefficient r_{k+1} was uniquely defined.

The arbitrariness that arises when constructing series (1.5) is equivalent to specifying the conditions

$$u(t,\vartheta)|_{\vartheta=0} = u^0(t), \quad T(t,\vartheta)|_{\vartheta=0} = T^0(t), \quad T_{\vartheta}(t,\vartheta)|_{\vartheta=0} = T^1(t) \tag{1.14}$$

with arbitrary functions of the right-hand sides, but in this case

$$u^0(0) = u_{00}, \quad T^0(0) = T_{00}, \quad T^1(0) = 0$$

Theorem. If the functions $u^0(t)$, $T^0(t)$, and $T(t)$, which occur in conditions (1.14), are analytical in a certain neighbourhood of the point $t = 0$, then series (1.5) converges in a certain neighbourhood of the point ($t = 0$, $\vartheta = 0$).

To prove this, problem (1.4), (1.6), (1.14) is reduced, using a well-known method [5], to a certain standard form, for which the analogue of Kovalevskaya's theorem on the existence and uniqueness of the solution in the class of analytical functions holds [5]. To refine the region of convergence of the series (and first of all when $\vartheta \geqslant 0$ when describing the gas compression) the functions r_{k+1}, u_k and T_k are analysed in detail.

Lemma. The coefficients r_{k+1}, u_k and T_k, when $k \geqslant 1$, are polynomials of ϑ and $e^{\vartheta/2}$, and the maximum total degree of the monomials of the form $\vartheta^h e^{m\vartheta/2}$ occurring the them does not exceed $2k$, i.e. $n + m/2 \leqslant 2k$. In this case each of the functions r_{k+1}, u_k and T_k necessarily contains a monomial of the form $e^{2k\vartheta}$ with a non-zero coefficient.

The proof is carried out by induction with respect to k using formulae (1.11)–(1.13) and in its main features repeats the corresponding proofs in the case of a non-heat-conducting gas (see, for example, [5, 8]), and hence will not be given here.

Using the lemma it can be established that the region of convergence of series (1.5), which solves problem (1.4), (1.6), (1.14), is given by the formula

$$Me^{2\vartheta}|t| < 1, \qquad M = \text{const} > 0 \tag{1.15}$$

i.e. this region is unbounded with respect to the variable ϑ

Taking formulae (1.9) and (1.10) into account, series (1.5) can be written in the form

$$\vartheta = (\mp\sqrt{\gamma/T_{00}}\,u_{00} - 1) \pm \sqrt{\gamma/T_{00}}\,(r-1)/t + tf(t,\vartheta)$$

$$u = (\pm\sqrt{\gamma^{-1}T_{00}}\,\vartheta + u_{00}) + tg(t,\vartheta), \qquad T = T_{00} + th(t,\vartheta) \tag{1.16}$$

where the functions $f(t, \vartheta)$, $g(t, \vartheta)$, $h(t, \vartheta)$ are analytical in the region (1.15). By the theorem on the existence of an implicitly specified function, the first of relations (1.16) defines ϑ as a function of the variables $(r-1)/t$ and t. Consequently, the two other gas-dynamic parameters u and T can also be assumed to be functions of these variables.

The Jacobian of the transition from the variables t' and r to the variables t' and ϑ can be written in the form

$$J = -r_\vartheta = \mp\sqrt{\gamma^{-1}T_{00}}\,t + t^2 q(t, \vartheta)$$

where the function $q(t, \vartheta)$ is also analytical in the region (1.15). Hence, when $t = 0$ the Jacobian vanishes, and for any finite value of $\vartheta_0 > 0$ a $t_0 > 0$ exists such that $J \neq 0$ when $|\vartheta| < \vartheta_0$ and $|t| < t_0$, $t \neq 0$.

2. APPLICATION OF THE SERIES

Series (1.5), which gives the solution of system (1.4), can be joined directly via the acoustic characteristic with the other solution that represents the flow of heat-conducting non-viscous gas. It can be shown that system (1.3) has two families of acoustic characteristics $C_\varkappa^\pm$, the trajectories of which $r = r_\pm(t)$ are solutions of the following differential equations

$$dr_\pm / dt = u \pm \sqrt{\gamma^{-1}T}$$

The initial data, specified on the $C_\varkappa^\pm$-characteristics, by virtue of the form of system (1.3), should be such that

$$\vartheta|_{C_\varkappa^\pm} = \varphi_1(t), \quad u|_{C_\varkappa^\pm} = \varphi_2(t), \quad T|_{C_\varkappa^\pm} = \varphi_3(t), \quad T_r|_{C_\varkappa^\pm} = \varphi_4(t) \tag{2.1}$$

The following additional constraint is imposed on the functions occurring on the right-hand side of relations (2.1)

$$a(\varphi_1(t), \varphi_3(t))\varphi_4(t) = \mp\sqrt{\gamma^{-1}\varphi_3(t)}\varphi_1'(t) - \varphi_2'(t) \mp \nu\sqrt{\gamma^{-1}\varphi_3(t)}\,\frac{\varphi_2(t)}{r_\pm(t)}$$

which is the necessary condition [5] for the characteristic problem (1.3), (2.1) to be solvable.

When it is necessary to join the analogue of the centred Riemann wave with a uniform gas at rest with parameters $\rho = 1$ and $T = 1$, the sonic $C_\varkappa^\pm$-characteristic is specified by the equation

$$r = 1 \pm t\gamma^{-1/2} \tag{2.2}$$

while the initial conditions on it are such that

$$\vartheta|_{C_\varkappa^\pm} = 0, \quad u|_{C_\varkappa^\pm} = 0, \quad T|_{C_\varkappa^\pm} = 1, \quad T_r|_{C_\varkappa^\pm} = 0 \tag{2.3}$$

These functions naturally satisfy the necessary condition for the characteristic Cauchy problem to be solvable.

In order that series (1.5), constructed above as a solution of problem (1.4), (1.6), (1.14), on characteristic (2.2) should be continuously joined with uniform rest (i.e. in order that conditions (2.3) should be satisfied), it is sufficient for the arbitrary constants occurring in series (1.5) to be taken such that the following equations are satisfied

$$T_{00} = 1, \qquad u_{00} = 0; \qquad T_k(0) = u_k(0) = 0, \qquad k \geqslant 1$$

Here relation (2.2) for the series (1.5) will be satisfied automatically, since it can be proved by induction with respect to k that in this case $r_{k+1}(0) = 0$ when $k \geqslant 1$.

We can also prove by the method employed earlier in [9], that in the case of the other flow (non-uniform rest) the arbitrary constants occurring in the coefficients of series (1.5), can be specified so that the flow, described by formula (1.5), is identical with this other flow on its sonic $C_\varkappa^\pm$-characteristic.

Consequently, the flow of a heat-conducting gas, constructed in the form of series (1.5) – the analogue of a centred wave – can be continuously joined with the other specified flow via the sonic characteristic.

If, for the flow specified by series (1.5), its sonic $C_\varkappa^\pm$-characteristic issues in the opposite direction of the change in time, then, necessarily after a certain time, its trajectory will lie in the region of convergence of series (1.5). Hence, on such a characteristic the flow will have no singularities, since the gas-dynamic parameters on it are specified by analytical functions. This in turn, by analogy with the case of a non-heat-conducting gas [14], enables us to formulate and solve the problem of obtaining the flow with a density distribution specified in advance, which continuously adjoins flow (1.5) via the above-mentioned sonic characteristic.

Using numerical methods similar to those used previously in [15] in the case of a non-heat-conducting gas, we will construct both flow (1.5) – a compression wave similar to a centred wave, and flows which continuously adjoin it from both sides. On the one hand, this is background flow which, in general, may not be uniform rest. On the other hand, this is flow, with a value of the density specified in advance, up to which compression of the gas layer also occurs. In this case, to calculate the flow in the neighbourhood of the point ($t = 0, r = 1$) it will be necessary to use formulae (1.16).

Using a well-known method [5] we can determine the asymptotic behaviour as $t \to -0$ of the gas parameters on a piston which produces a compression wave described using series (1.5).

Suppose the functions $r = R(t)$, $\vartheta = \Theta(t)$ and $u = U(t)$ specify the trajectory of motion of an impenetrable piston and the parameters of the gas on it respectively. Then, first

$$\frac{dR}{dt} = U(t)$$

and second

$$\sum_{k=0}^{\infty} r_k(\Theta(t))\frac{t^k}{k!} = R(t)$$

Differentiating the second relation with respect to t and taking the first relation into account, we obtain the following ordinary differential equation for the function $\Theta(t)$

$$\left[\sum_{k=0}^{\infty} r_{k\Theta}(\Theta(t))\frac{t^k}{k!}\right]\frac{d\Theta(t)}{dt} + \sum_{k=0}^{\infty} r_{k+1}(\Theta(t))\frac{t^k}{k!} = \sum_{k=0}^{\infty} u_k(\Theta(t))\frac{t^k}{k!} \tag{2.4}$$

If we retain the principal terms in the series occurring in Eq. (2.4), we obtain the equation

$$td\Theta(t)/dt = -1$$

the general solution of which is as follows:

$$\Theta(t) = -\ln(-t\rho_{01}^{-1}), \qquad \rho_{01} = \text{const} > 0, \qquad t < 0 \tag{2.5}$$

Naturally, formula (2.5) gives a certain approximation for the required relation $\Theta(t)$. When using this approximation for the value of the gas density on the compressing piston, the following approximate relation is obtained

$$\rho\,|_{r=R(t)} \approx (-t)^{-1}\rho_{01}, \qquad t < 0 \tag{2.6}$$

For numerical calculations of the intense compression of a plane layer of a heat-conducting gas [12], carried out using the "Tigr" software package, which is highly recommended for solving a wide range of applied problems, the following asymptotic relation is obtained for $-t \geqslant 10^{-6}$

$$\rho\,|_{r=R(t)} \approx (-t)^{-1.15}\rho_{01}, \qquad t < 0$$

which is fairly close to (2.6).

However, approximate relations (2.5) and (2.6) must be used with certain provisos. Relation (2.5) can be rewritten in the equivalent form

$$(-t)e^{\Theta} = \rho_{01}$$

The last relation establishes a relation between the variables t and θ. A comparison of this relation with the boundary of the region of convergence of series (1.5) leads to the following conclusion: if for any negative values of t the value of the function $\vartheta\,|_{r=R(t)}$ lies in the region (1.15), then, as $t \to -0$ these values will necessarily fall outside the limit of the region of convergence of series (1.5) and it will be inadmissible to use this series there.

This conclusion is also confirmed by the following consideration. If function (2.5) is substituted into the series which specifies the gas velocity, then, as $t \to -0$ the order of a term of the series $u_1(\Theta)t$ will turn out to be (due to the term $te^{2\Theta}$) greater than the order of the zeroth term of the series $u_0(\Theta)$. This in turn indicates that the series obtained in this way is not asymptotic.

Consequently, using series (1.5) one cannot give a strict mathematical proof of the possibility of compressing a non-zero mass of gas to infinite density (as was done in [5] in the case of non-heat-conducting gas). However, the fact that there is no limit with respect to the variable ϑ of the region of convergence of this series leads to the following mathematically rigorously based conclusion: for any density $\rho_1 > 1$, specified in advance, a non-zero mass of a uniform gas at rest (with $\rho = 1$) exists which, due to the action of an impenetrable piston, can be compressed shock-free to a density ρ_1. This conclusion follows from the fact that for any value of $\rho_1 > 1$ one can choose a trajectory

$$r = R(t) = \sum_{k=0}^{\infty} r_k(\Theta(t))\frac{t^k}{k!}$$

of the motion of the compressing piston, which, for all values of $0 \leqslant \Theta \leqslant \vartheta_1 = \ln(\rho_1)$, does not depart from the region (1.15), i.e. from the region of convergence of series (1.5). Then, the value of $t_1 - t_0$ specifies the initial width of the layer of gas with $\rho = 1$, compressed to $\rho = \rho_1$. Here the instants of time t_1 and t_0 are such that $\Theta(t_1) = \vartheta_1$, $\Theta(t_0) = 0$.

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